Bernoulli's Equation

Any Equation in the form of

$$\frac{dy}{dx} + Py = Qy^n$$, where P and Q are function of x only

divide whole equation by y^n

$$\frac{1}{y^n}\frac{dy}{dx} + P\frac{y}{y^n} = Q$$

$$\frac{1}{y^n}\frac{dy}{dx} + Py^{1-n} = Q$$

Put $y^{1-n} = z$

differenciate both sides w.r.t x

$$(1-n)\,y^{1-n-1}\frac{dy}{dx} = \frac{dz}{dx}$$

$$(1-n)\,y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$$

$$y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dz}{dx}$$

$$\frac{1}{1-n}\frac{dz}{dx} + Pz = Q$$, linear in z

I.F $e^{\int P\,dx}$

solution is

$$z \cdot I.F = \int I.F \cdot Q\,dx + C$$

Replace z by y and get Solution.

Solve for P, x, y and Claraut Equation

when order of equation is one and degree is more than one, than use solve for P, x, y

for P, Change equation in quadratic in P and Solve this

for y. If equation can be explained equal to y

for x. If equation can be explained in proper form equal to x.

Claraut Equation :- If equation in Particular form of

$$y = Px + f(P)$$

then replace P by c and Solution is

$$y = cx + f(c)$$

Q $P = \sin(y - px)$

$y - px = \sin^{-1} p$

$y = px + \sin^{-1} p$

This is clairaut form. Putting c for P and Solution is $y = cx + \sin^{-1} c$

Q Reduce the equation $y^2 (y - xp) = x^4 p^2$ to clairaut equation and find solution.

Soln.

Put $x = \frac{1}{X}$ $y = \frac{1}{Y}$

$dx = -\frac{1}{X^2} dX$ $dy = -\frac{1}{Y^2} dY$

$$P = \frac{dy}{dx} = -\frac{1}{Y^2} \frac{dY}{dX} (-X^2) = \frac{X^2}{Y^2} \frac{dY}{dX} = \frac{X^2}{Y^2} P$$

Put value of P in Given equation

$$\left(\frac{1}{Y}\right)^2 \left(\frac{1}{Y} - \frac{1}{X} \cdot \frac{X^2}{Y^2} P\right) = \frac{1}{X^4} \left(\frac{X^2}{Y^2} P\right)^2$$

$$\frac{1}{Y^2} \left(\frac{1}{Y} - \frac{X}{Y^2} P\right) = \frac{1}{X^4} \frac{X^4}{Y^4} P^2$$

$$\frac{1}{\cancel{Y^2}} (Y - PX) = \frac{P^2}{\cancel{Y^4}}$$

$\Rightarrow Y = PX + P^2$, this is form of Clairaut equation

Put $P = c$ and Solution is

$Y = cX + c^2$

$\frac{1}{y} = \frac{c}{x} + c^2$ or $x = cy + c^2 xy$